# Navigation and control problems during low-thrust transfer from Earth to Jupiter

E.L. Akim, V.V. Sazonov,
V.A. Stepanyants, A.G. Tuchin

**The problem:**
High-level nongravitation accelerations as a result of random variations in the thrust of SEP (solar electric propulsion)

**The purposes of the work:**
Analysis of control capability for the electric propulsion spacecraft.

# Introduction

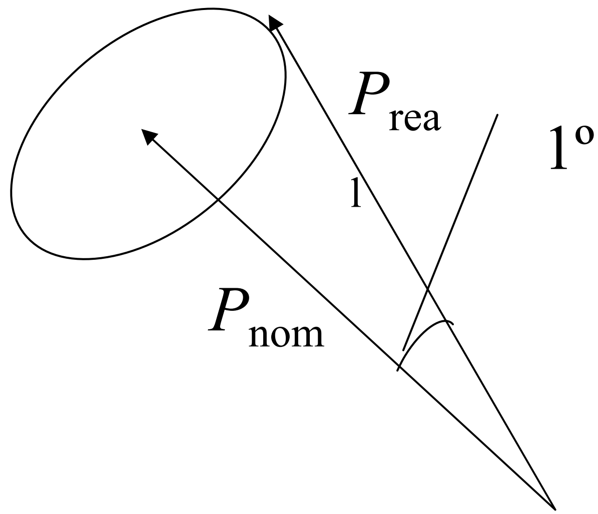
- The nominal trajectories of the flight were obtained by M.S. Konstantinov, G.G. Fedotov, and V.G. Petukhov from Research Institute of Applied Mechanics and Electrodynamics of Moscow Aviation Institute.
- The Earth gravity assist is used to fly to Jupiter. The trajectory consists of two phases. The duration of the first phase "Earth-Earth" equals 450 days. The duration of the second phase "Earth-Jupiter" equals 1030 days.
- The solar electric propulsion (SEP) of the type SPT-140 are used.
- $V_{\text{infinity}} \sim 5000$ m/s (4848 m/s, 5544 m/s).
- Final mass of SC after SEP separation is approximately 4000 kg.

# The purposes of the analysis

- How do errors of the SEP operation affect the precision of the orbit determination, the instant of the SC arrival to Jupiter, and the propellant consumption?
- How often must the correction of the SEP control program be carried out?
- What are requirements for on-board and ground-based trajectory measurements? (specifically for accuracy).

The analysis was performed only for the part of the flight after the Earth flyby.

# SEP errors




The current nominal thrust is

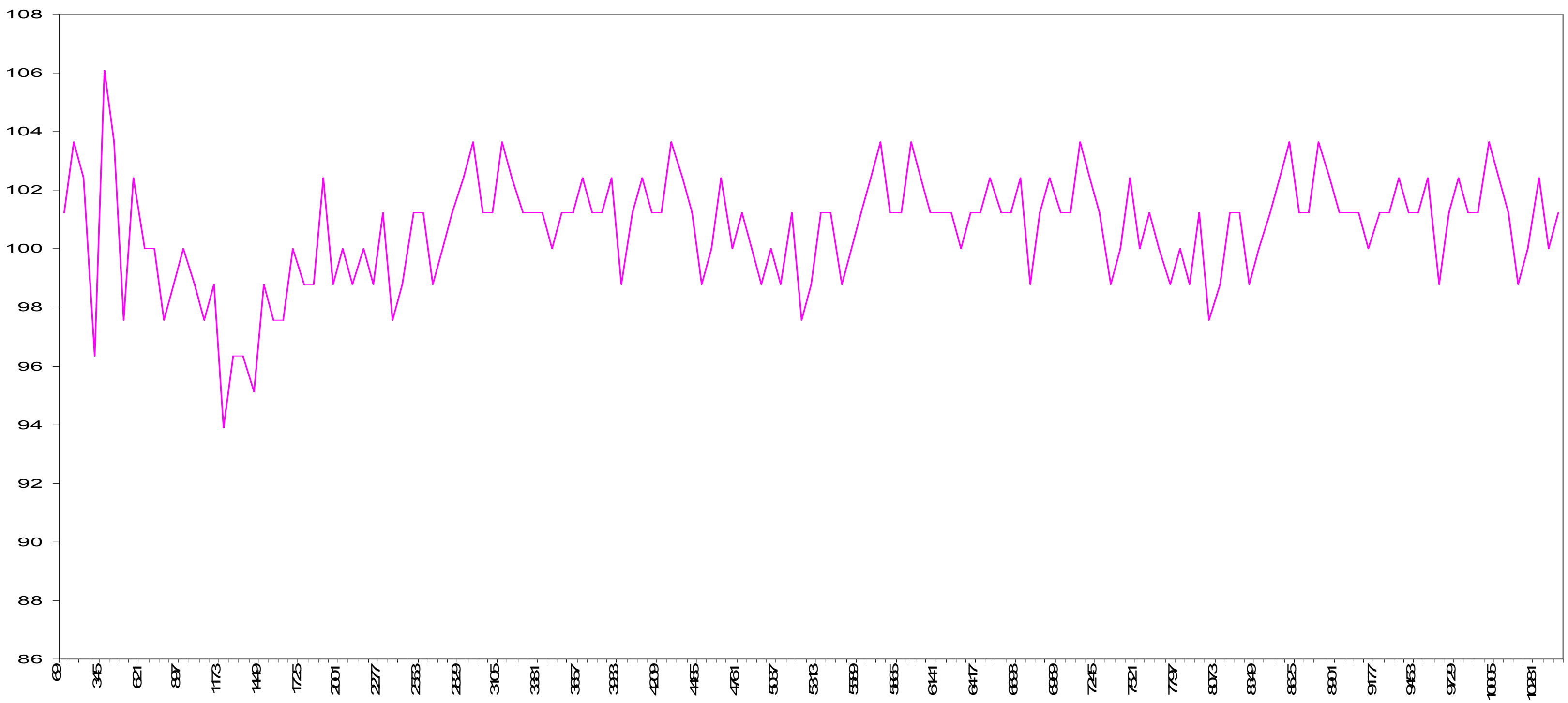
$$P_{nom} = \frac{1-\beta t}{r^{1.7}} P_0$$

Here, $P_0$ is the initial thrust magnitude without degradation of the solar battery at the distance 1 AU from Sun;
$r$ is the distance from Sun in AU;
$\beta$ is the degradation rate of the solar battery, $\beta \cong 8.2\ 10^{-5}$ day$^{-1}$.

The relative error of thrust magnitude is less than 6%.
The direction error of thrust vector (pointing angle) is less than 1°.

# The random variations of the thrust process

- The deviation of the thrust vector from its nominal value is assumed to be the sum of the systematic bias and the noise.
- The systematic bias has a correlation time constant of 5 days.
- The noise has a correlation time constant of 0.5 days.
- RMS of the noise equals about 30% of total RMS.



# Equations of the optimal SC motion

$$\dot{\mathbf{r}} = \mathbf{v}, \quad \dot{\mathbf{v}} = -\frac{\mathbf{r}}{r^3} + \frac{a_0(1-\beta t)}{r^\alpha \xi}\mathbf{e}, \quad \dot{\xi} = -\frac{q(1-\beta t)}{r^\alpha}, \quad \mathbf{e} = \frac{\mathbf{p}_v}{p_v} \quad (t_0 \le t \le T)$$

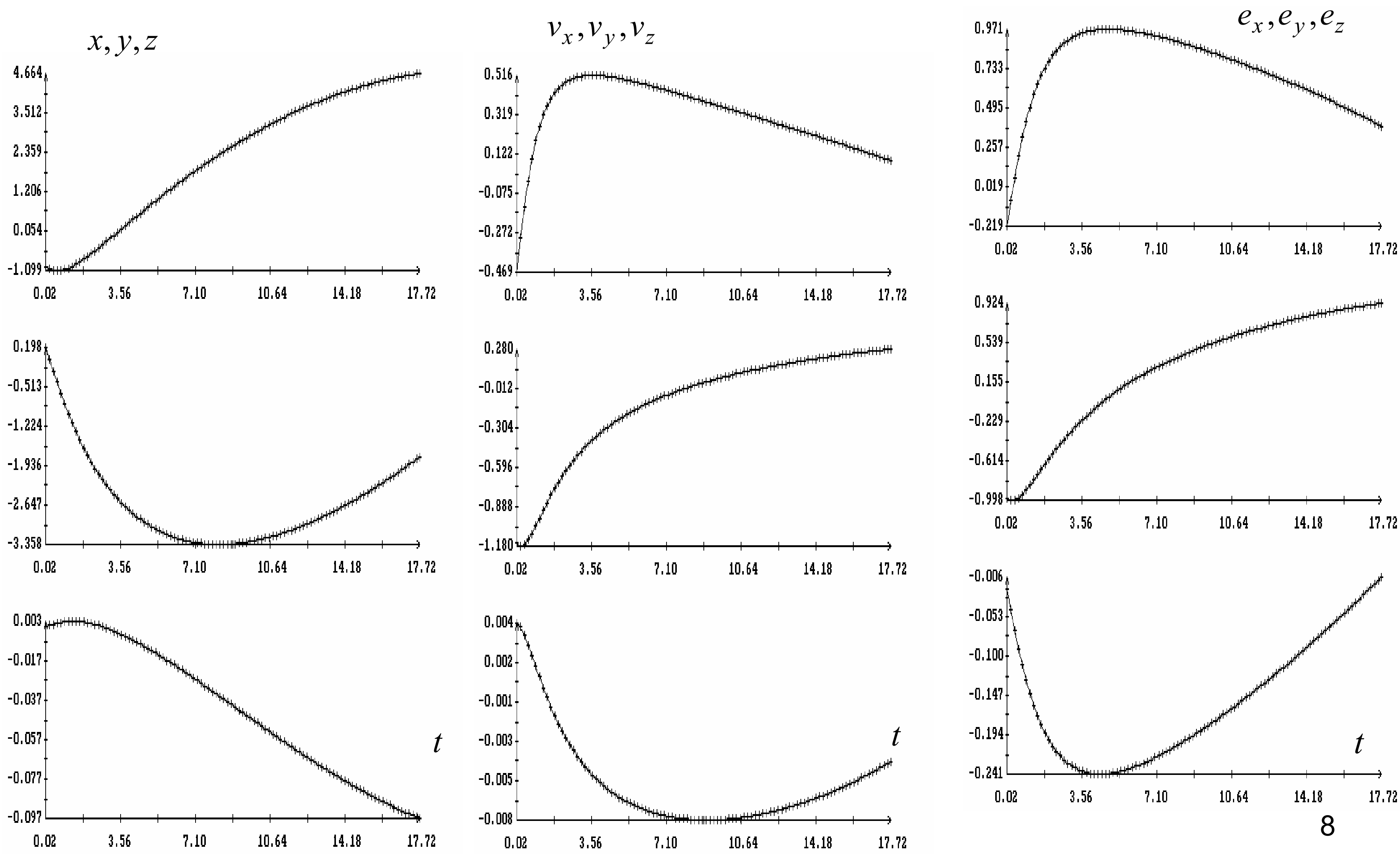
$$\dot{\mathbf{p}}_r = \frac{1}{r^3}\left[\mathbf{p}_v - \frac{3(\mathbf{p}_v \cdot \mathbf{r})\mathbf{r}}{r^2}\right] + \frac{\alpha a_0 (1-\beta t)}{r^{\alpha+2}}\left(\frac{p_v}{\xi} - q p_\xi\right)\mathbf{r}, \quad \dot{\mathbf{p}}_v = -\mathbf{p}_r, \quad \dot{p}_\xi = \frac{(1-\beta t) p_v}{r^\alpha \xi^2}$$

$$r = |\mathbf{r}|, \quad p_v = |\mathbf{p}_v|, \quad \xi = \frac{m}{m_0}, \quad a_0 = \frac{P_0 A^2}{m_0 \mu}, \quad q = \frac{a_0}{P_s g_0}\sqrt{\frac{\mu}{A}}$$

$$a_0 = 0.0314, \quad \alpha = 1.7, \quad \beta = 0.00477, \quad q = 0.0325$$

# Approximation of the arc of the nominal optimal trajectory after perturbation maneuver

# The least squares method was used, $\sigma = 3 \cdot 10^{-5}$



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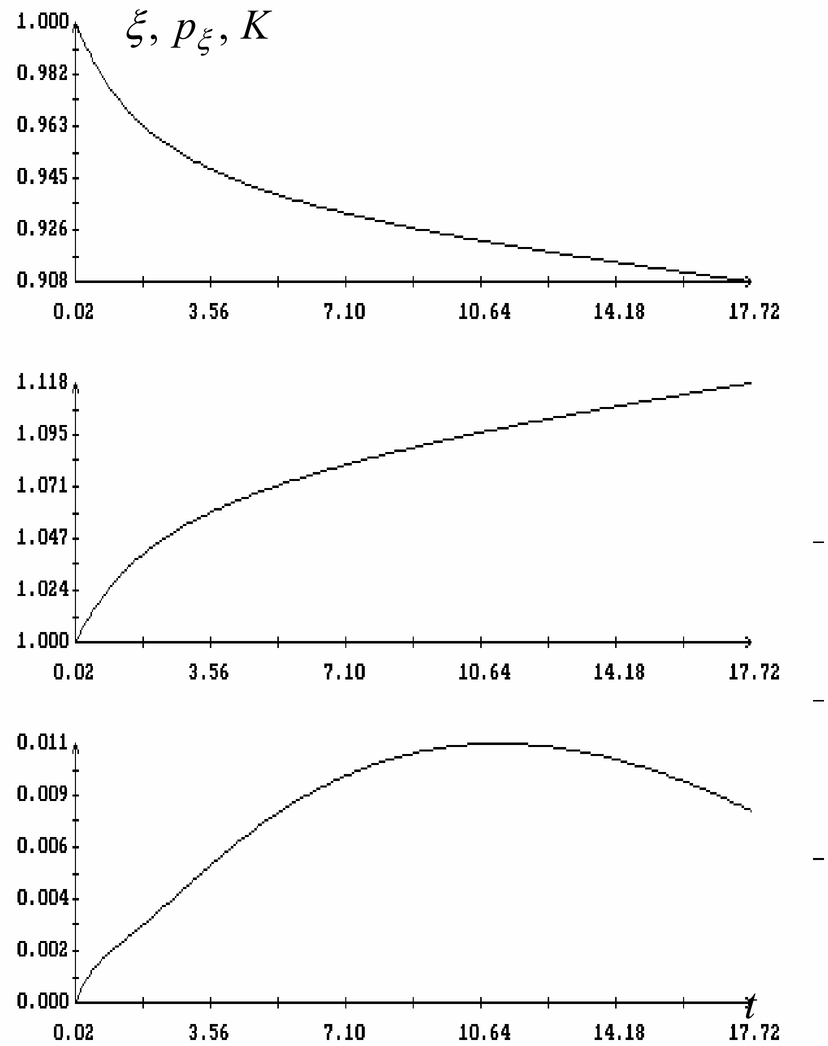
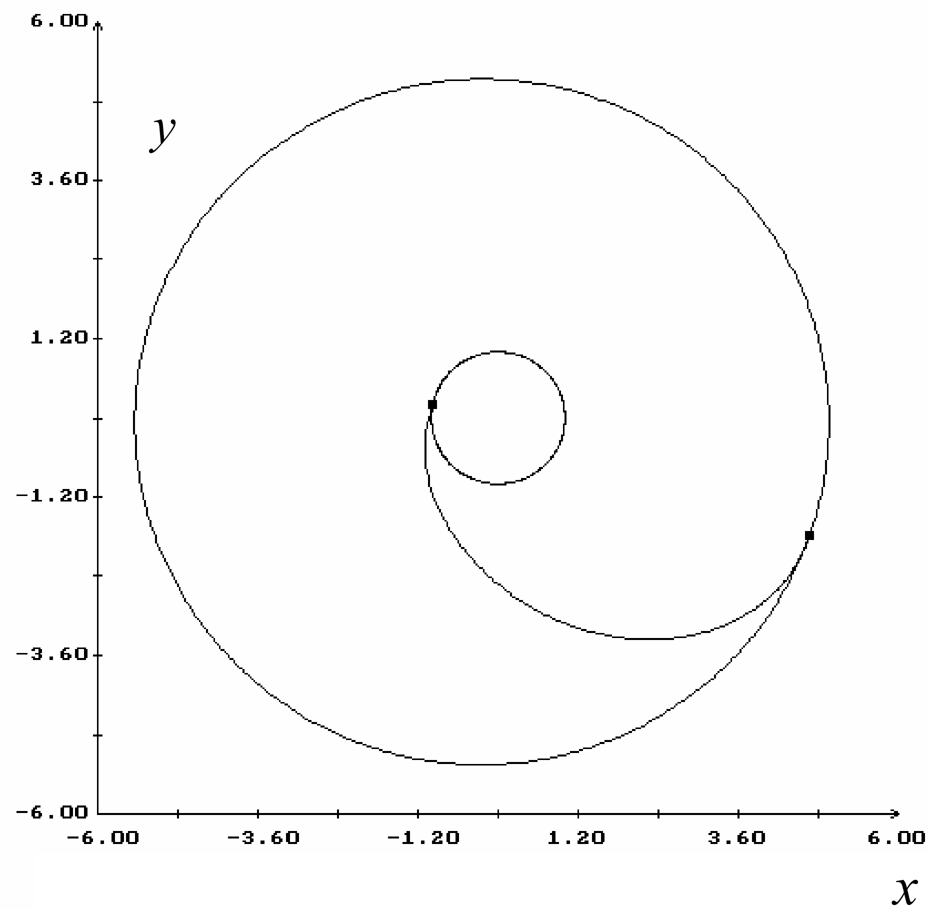



astronomical unit

t=2·π·FlightDay/365.25

# Variational problems for the hit of Jupiter

Functional $F = 1 - \xi(T) + \dfrac{|\mathbf{v}(T) - \mathbf{v}_J(\tau + T)|^2}{2V^2}$, $V = 0.377\ (11.2\,\text{km/s})$

Boundary conditions for the case of prescribed $T$:

$\mathbf{r}(0)$ and $\mathbf{v}(0)$ are given, $\xi(0) = p_\xi(0) = 1$, $p_\xi(T) \geq 0$

$$\mathbf{r}(T) = \mathbf{r}_J(\tau + T), \quad \mathbf{p}_v(T) + \frac{a_0}{V^2} p_\xi(T)[\mathbf{v}(T) - \mathbf{v}_J(\tau + T)] = 0$$

If $T$ is a varied quantity then the above conditions have to be enlarged

by the relation

$$\mathbf{p}_r(T) \cdot [\mathbf{v}(T) - \mathbf{v}_J(\tau + T)] + \frac{a_0}{r^\alpha(T)} \left( \frac{p_v(T)}{\xi(T)} - q p_\xi(T) \right) = 0$$

# Optimal trajectory for $t_0 = 0.02$ (1.167d) and fixed $T = 17.72$ (1030d)

$$\xi(T) = 0.9078,\ |\mathbf{v}(T) - \mathbf{v}_J(\tau + T)| = 0.1628\,(4.85\ \text{km/s}),\ F = 0.1855$$

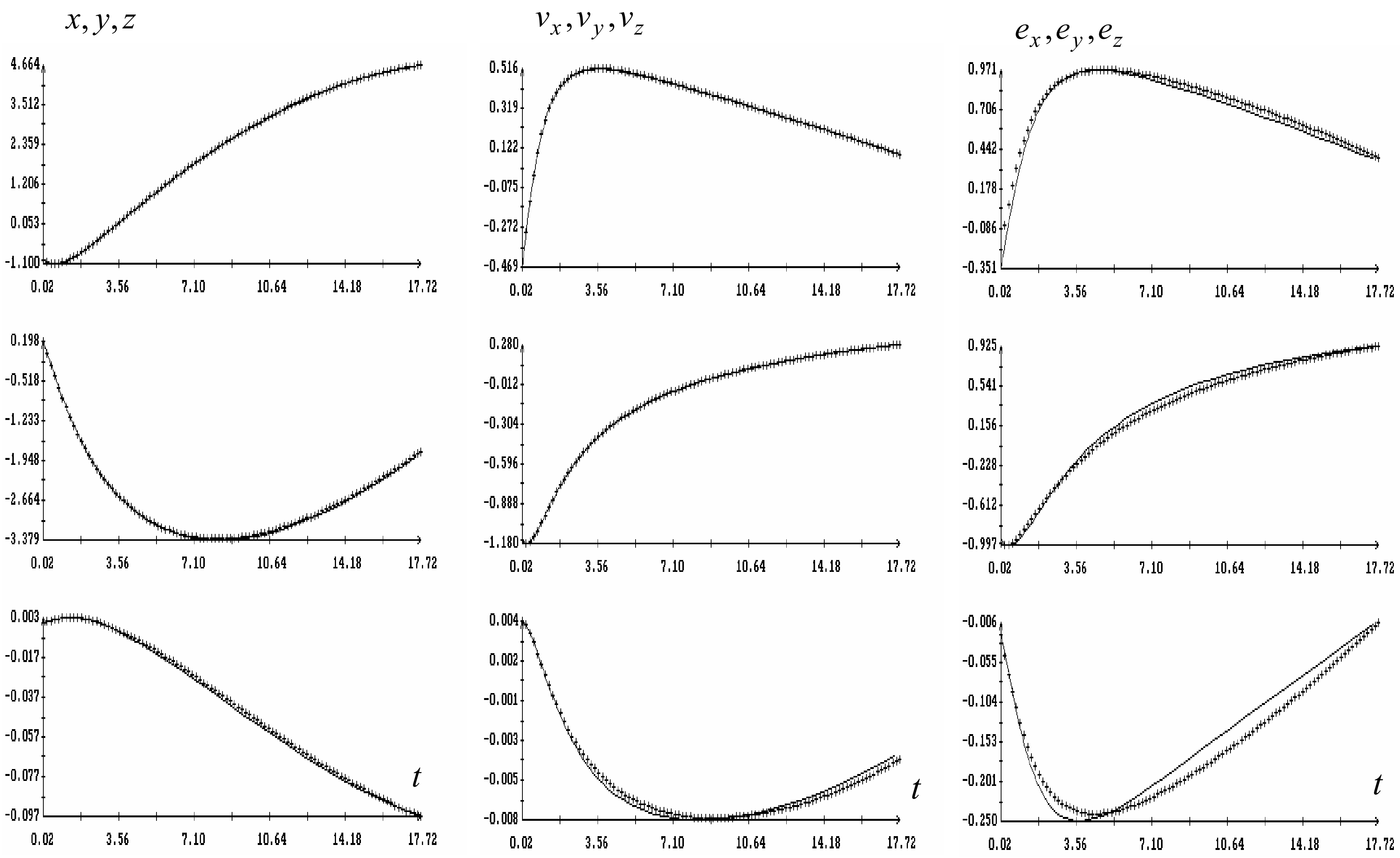
$$\tau + T \rightarrow 03.01.2022,\quad \sigma = 9.6 \cdot 10^{-4}$$

Optimal trajectory for $t_0 = 0.02\,(1.167\,\text{d})$ and varied $T$:

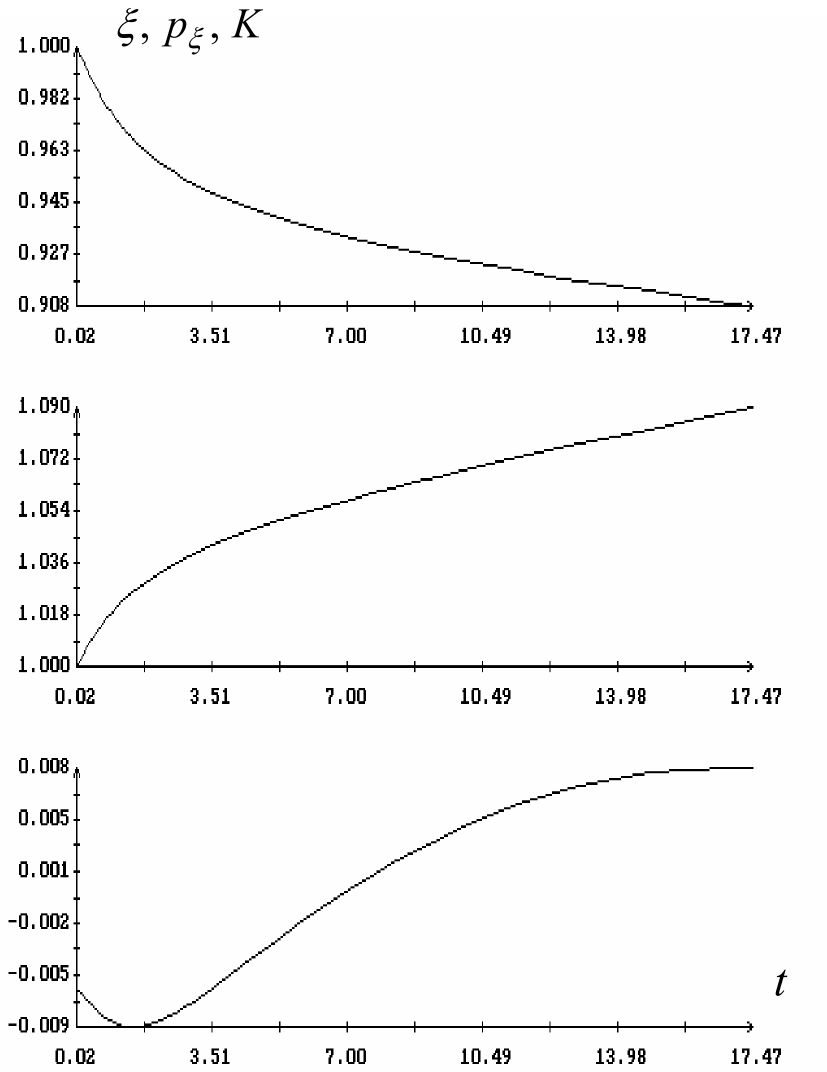
$$\xi(T) = 0.9085,\ |\mathbf{v}(T) - \mathbf{v}_J(\tau + T)| = 0.1631(4.86\ \text{km/s}),\ F = 0.1852$$

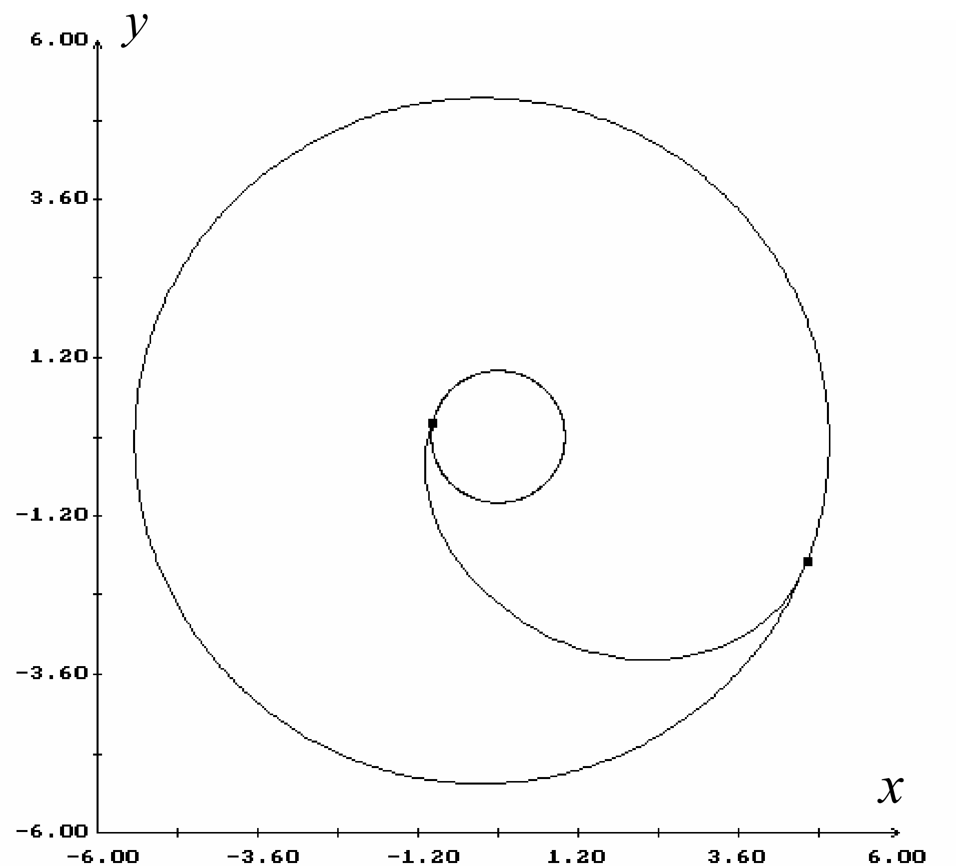
$$T = 17.47\,(1016\,\text{d}),\ \tau + T \rightarrow 20.12.2021$$

# The optimal trajectory with varied $T$



# The optimal trajectory with varied $T$



# Variational problems for the approach to the Jupiter sphere of influence

The functional and the boundary conditions are formulated in much the same as before, but two relations should be modified:

$$\mathbf{r}(T) = \mathbf{r}_J(\tau + T) + \rho(Z\boldsymbol{\tau} + X\mathbf{n} + Y\mathbf{b})$$

$$\mathbf{p}_r(T)\cdot[\mathbf{v}(T) - \mathbf{v}_J(t_0 + T) - \Omega\rho(Z\mathbf{n} - X\boldsymbol{\tau})] + \frac{a_0}{r^{\alpha}(T)}\left(\frac{p_v(T)}{\xi(T)} - qp_{\xi}(T)\right) = 0$$

Here, the quantities

$$\boldsymbol{\tau} = \frac{\mathbf{v}_J}{|\mathbf{v}_J|}, \quad \mathbf{b} = \frac{\mathbf{r}_J \times \mathbf{v}_J}{|\mathbf{r}_J \times \mathbf{v}_J|}, \quad \mathbf{n} = \mathbf{b} \times \boldsymbol{\tau}, \quad \Omega = \frac{|\mathbf{r}_J \times \mathbf{v}_J|}{|\mathbf{r}_J|^3 \cdot |\mathbf{v}_J|^2}$$

are calculated at the point $\tau + T$,

$$\rho = 0.322, \quad Z = \sqrt{1 - X^2 - Y^2}, \quad X^2 + Y^2 < 1$$

# Examples of flyby hyperbolas

$$X = 0.0984113,\ \ Y = 0.0060593,\ \ T = 15.996\ (929.8\ \mathrm{d}),\ \ F = 0.1841,\ \ \xi(T) = 0.9109$$
$$|\,\mathbf{v}(T) - \mathbf{v}_J(\tau + T)\,| = 0.1643\ (4.89\,\mathrm{km/s}),\ \ r_\pi = 72004\,\mathrm{km},\ \ e = 1.01062,\ \ i = 0.358°$$

$$X = 0.0961,\ \ Y = 0.019,\ \ T = 16.004\ (930.3\ \mathrm{d}),\ \ F = 0.1840,\ \ \xi(T) = 0.9109$$
$$|\,\mathbf{v}(T) - \mathbf{v}_J(\tau + T)\,| = 0.1643\ (4.89\,\mathrm{km/s}),\ \ r_\pi = 100730\,\mathrm{km},\ \ e = 1.01485,\ \ i = 40.934°$$

$$X = 0.0890,\ \ Y = 0.0256,\ \ T = 15.989\ (929.5\ \mathrm{d}),\ \ F = 0.1842,\ \ \xi(T) = 0.9109$$
$$|\,\mathbf{v}(T) - \mathbf{v}_J(\tau + T)\,| = 0.1645\ (4.90\,\mathrm{km/s}),\ \ r_\pi = 100800\,\mathrm{km},\ \ e = 1.01491,\ \ i = 76.697°$$

$$X = 0.15,\ \ Y = 0,\ \ T = 16.14\ (936.3\ \mathrm{d}),\ \ F = 0.1824,\ \ \xi(T) = 0.9105$$
$$|\,\mathbf{v}(T) - \mathbf{v}_J(\tau + T)\,| = 0.1625\ (4.84\,\mathrm{km/s}),\ \ r_\pi = 2.07 \cdot 10^6\ \mathrm{km},\ \ e = 1.2967,\ \ i = 18.664°$$

# Trajectory measurements by ground stations Assumptions

- Two Russian ground stations are used: Ussurijsk and Bear Lakes.
- Two-way X-band (8.4 Ghz) Doppler and range data are acquired from these stations.
- The measurement errors (3σ) equal 0.2 mm/s for two-way Doppler and 20 m for range.
- Measurements are carried out three times per day with a batch size of 20 min. Depending on visibility conditions, they performed two times from Ussurijsk and once from Bear Lakes or once  from Ussurijsk and two times from Bear Lakes.

# The on-board measurements. Assumptions

- High-accuracy accelerometers of the on-board strap-down inertial system (SDINS) are used for the orbit determination.
- Measurements by SDINS are used for delta-V on-board evaluation. Delta-V vector has three components: one component is directed along the thrust vector, two another components lays in the plane that is orthogonal to the thrust vector.
- Measurements are transmitted four times per day to Earth and used for the orbit determination.
- Three levels of accelerometers errors ($1\sigma$) are considered: $10^{-7}$ m/s$^2$, $5 \cdot 10^{-7}$ m/s$^2$, $10^{-6}$ m/s$^2$.

# The control of the flight after the Earth flyby

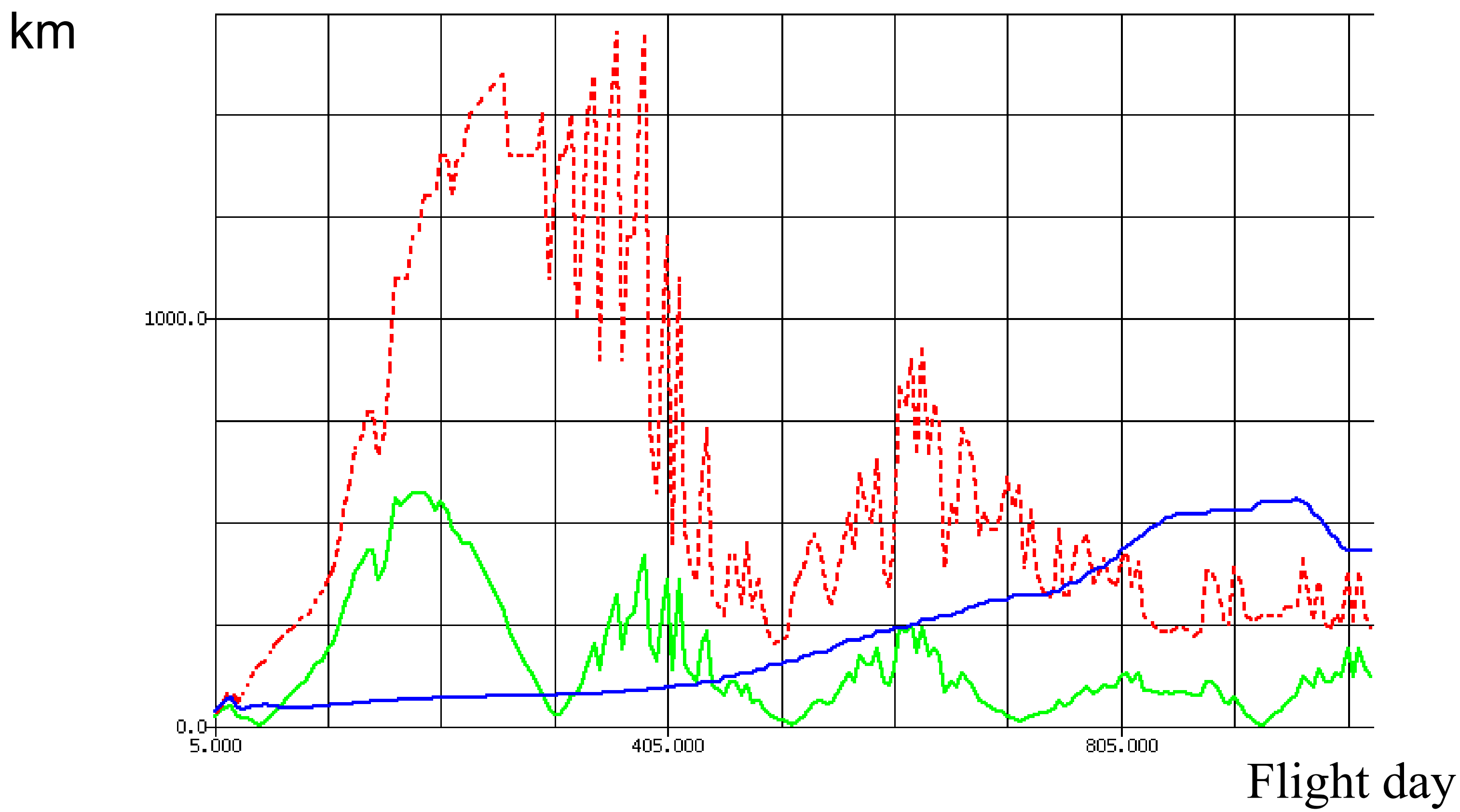
- The on-board computer controls the thrust direction according to the control law transmitted from Earth.
- The SC orbit determination is implemented by the Mission control center on Earth using ground and on-board measurement data.
- The control law in the residuary part of the flight is adjusted according to orbit determination results.
- The control law can be modified each 10 days when needed.

# The orbit determination modeling. Covariance analysis

- If only ground trajectory measurements are available then extended SC state vector is used. This extended vector includes coordinates of SC, its velocity, and parameters of the SEP model.
- If extra accelerometer measurements are available then they are taken into account to modify the motion model.
- The covariance analysis was implemented for both these cases. Results are presented on the following slides.

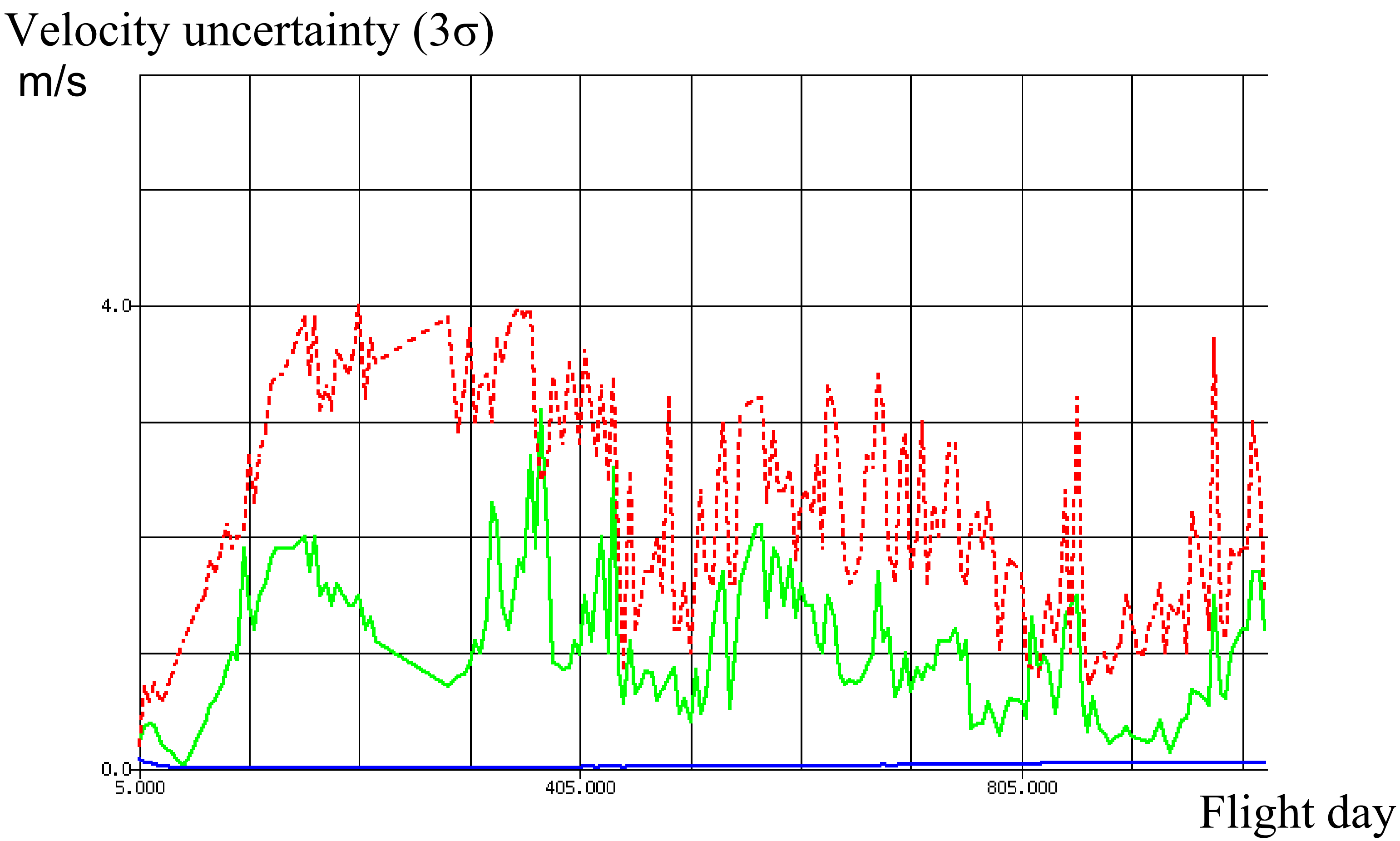
# Two-way Doppler and range data from ground station are available only

Position uncertainty (3σ)



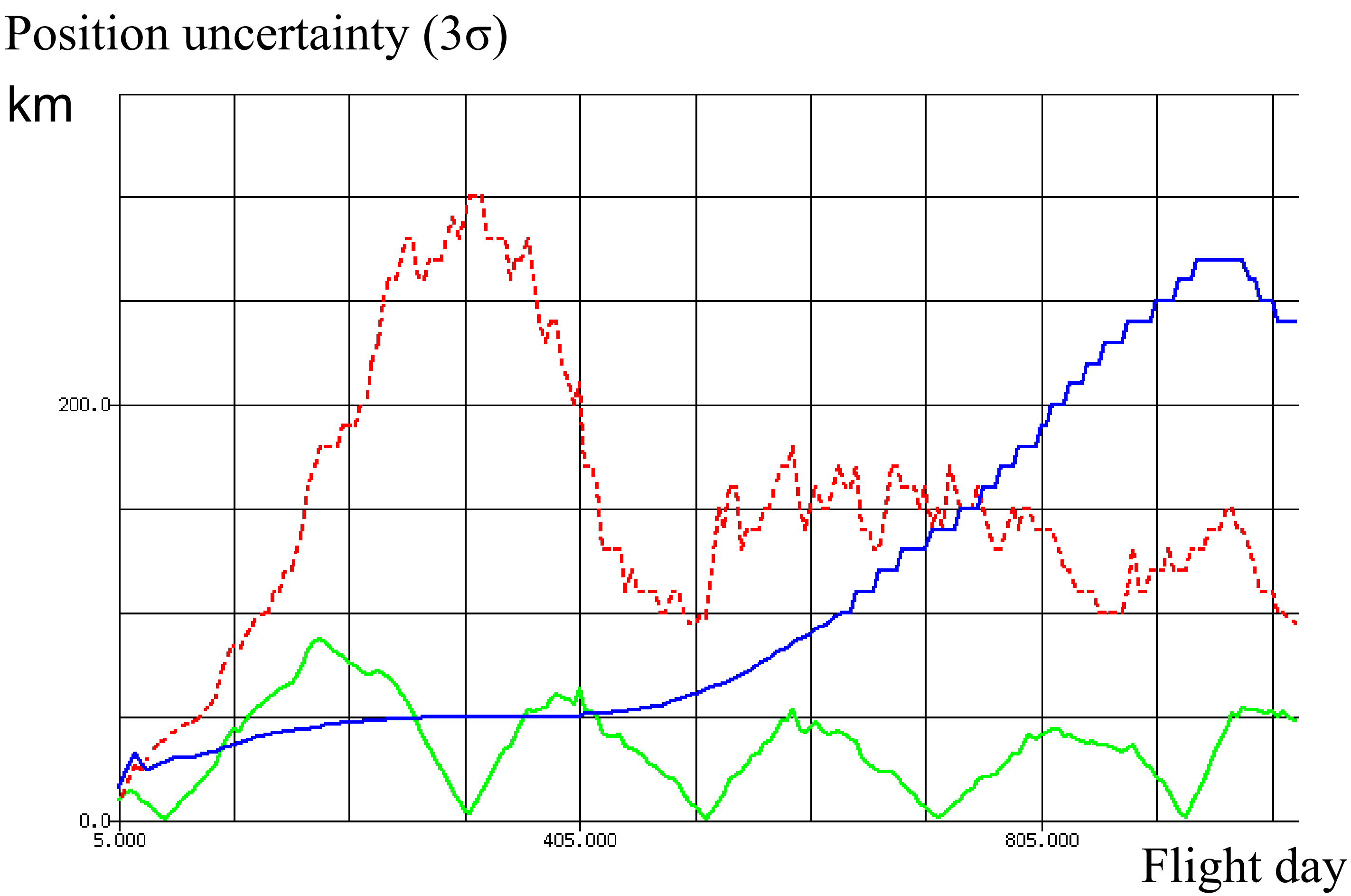

Red plot depicts the error along the orbit transversal (T), green and blue plots represent the radial error (R) and the error along the normal to the orbital plane (B). Maximal values of the errors for R,T,B are 570, 1700, 560 km.

# Two-way Doppler and range data from ground station are available only




Red plot depicts the error along the orbit transversal, green and blue plots represent the radial error and the error along the normal to the orbital plane. Maximal values of the errors for R,T,B are 3.1, 4.1, 0.1 m/s.

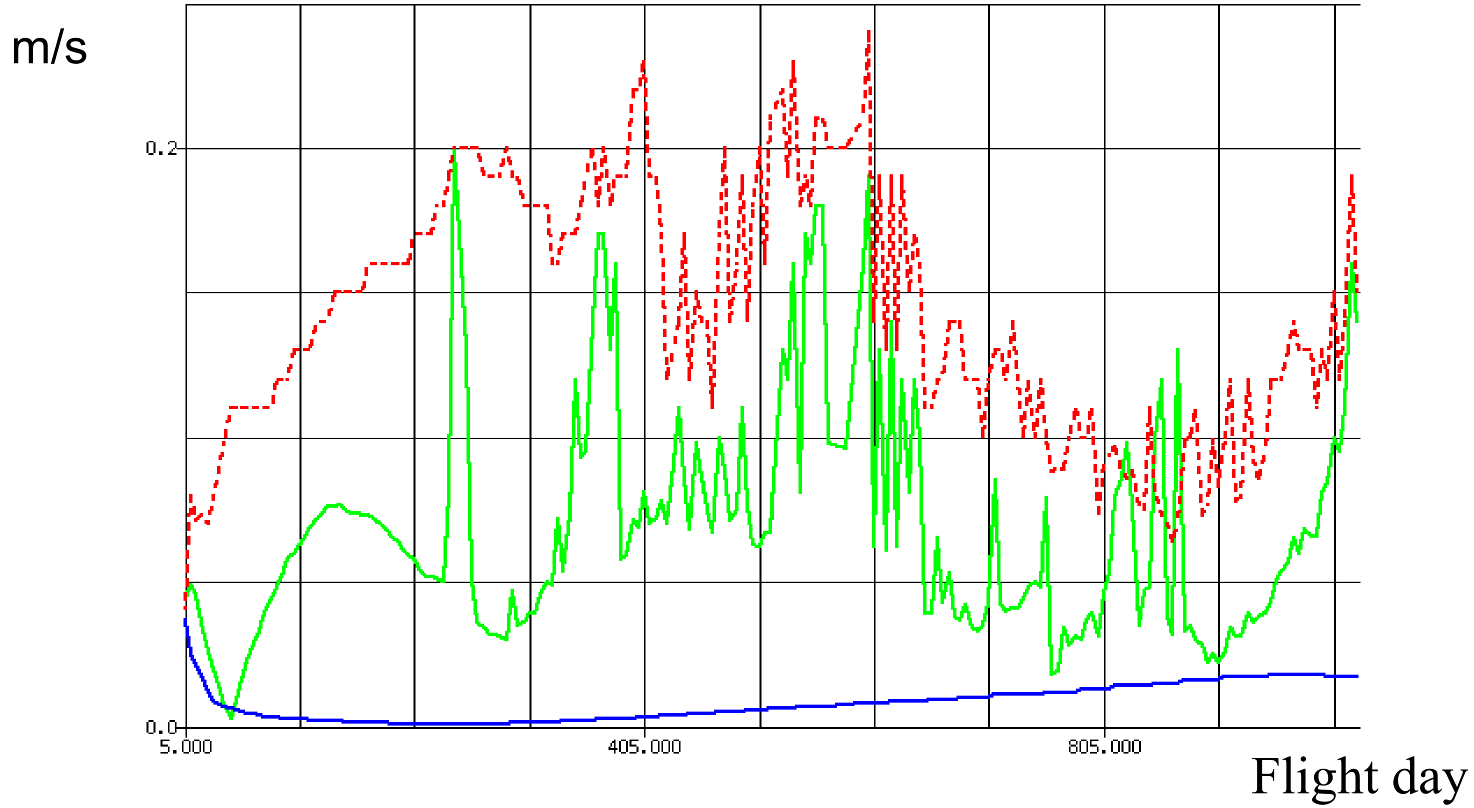
# Two-way Doppler and range data from ground station as well as on-board accelerometers ($\sigma = 10^{-7}$ m/s$^2$) are available




Red plot depicts the error along the orbit transversal, green and blue plots represent the radial error and the error along the normal to the orbital plane. Maximal values of the errors for R,T,B are 87, 300, 270 km.

# Two-way Doppler and range data from ground station as well as on-board accelerometers ($\sigma = 10^{-7}$ m/s$^2$) are available

Velocity uncertainty (3σ)




Red plot depicts the error along the orbit transversal, green and blue plots represent the radial error and the error along the normal to the orbital plane. Maximal values of the errors for R,T,B are 0.2, 0.25, 0.04 m/s.

# The estimates of the date of the arrival to Jupiter and additional propellant consumption

- The SC motion is simulated for the first 10 flight days according to the nominal control law and the error thrust model.
- Then we disturb the SC state vector adding to it navigation errors and evaluate the new control law for residuary part of the flight.
- We repeat the simulation for the next 10 flight days using the new control law and the error thrust model, disturb the SC state vector obtained, and so on until the arrival to the Jupiter sphere of influence.
- We repeated calculations for a lot values of thrust errors (Monte-Carlo method was used).
- Thus we estimated the scatter of the arrival date and additional propellant consumption.

# SC arrival errors at the Jupiter sphere of influence

| Variant | Position, km | Velocity, m/s |
|---|---|---|
| without accelerometers | 1410 | 2.3 |
| using accelerometers, $\sigma = 10^{-7}$ m/s$^2$ | 990 | 1.2 |
| using accelerometers, $\sigma = 5\ 10^{-7}$ m/s$^2$ | 1000 | 1.2 |
| using accelerometers, $\sigma = 10^{-6}$ m/s$^2$ | 1010 | 1.2 |

# The contributions of navigation errors to SC arrival errors at the Jupiter sphere of influence

| Variant | Position, km | Velocity, m/s |
|---|---|---|
| without accelerometers | 1045 | 2.0 |
| using accelerometers, $\sigma = 10^{-7}$ m/s$^2$ | 290 | 0.38 |
| using accelerometers, $\sigma = 5\ 10^{-7}$ m/s$^2$ | 320 | 0.43 |
| using accelerometers, $\sigma = 10^{-6}$ m/s$^2$ | 340 | 0.47 |

# The influence of thrust errors on the arrival date and propellant consumption

- The date of the arrival to the Jupiter sphere of influence has the variation of $\pm 25$ days.
- The compensation of thrust errors requires the reserve of propellant that is about 1% of its total mass.

# Conclusion

- The SC arrival errors ($3\sigma$) at the Jupiter sphere of influence are less than 1410 km for position and 2.3 m/s for velocity.
- The control law should be modified periodically. Updating every 10 days is sufficient.
- The SC control in the vicinity of Jupiter must be performed by chemical thrusters.
- Two Russian ground stations in X-band (Ussurijsk and Bear Lakes) are used. Their measurement errors ($3\sigma$) equal 0.2 mm/s for two-way Doppler and 20 m for range.
- On-board accelerometers enable decreasing errors of the orbit determination and errors of the arrival to the Jupiter sphere of influence.