Моделирование различных типов связи в ассоциациях нейроподобных осцилляторов

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Диффузионная связь

$$\dot{u}_j = D(u_{j-1} - 2u_j + u_{j+1}) + F(u_j), \quad j = 1, \ldots, n$$

$$\dot{u} = F(u),$$

$$u_*(t) \equiv u_*(t + T)$$

$$\dot{u}_j = \varepsilon D(u_{j-1} - 2u_j + u_{j+1}) + (A_0 + \varepsilon A_1)u_j + F_2(u_j, u_j) + F_3(u_j, u_j, u_j) + \ldots,$$

$$j = 1, \ldots, n$$

$$u_0 =?, \quad u_{n+1} =?$$

$$A_0 a = i\omega a, \quad A_0^T b = -i\omega b$$

Нормальная форма

$$u_j(t,\varepsilon) = \sqrt{\varepsilon}u_{0j}(t,s) + \varepsilon u_{1j}(t,s) + \varepsilon^{3/2}u_{2j}(t,s) + \dots$$

$$s = \varepsilon t,$$

$$u_{0j}(t,s) = z_j(s)\exp(i\omega t)a + \bar{z}_j(s)\exp(-i\omega t)\bar{a}$$

$$z_j' = (A_1 a, b)z_j + (d_0 + ic_0)|z_j|^2 z_j + (Da, b)(z_{j-1} - 2z_j + z_{j+1}), \quad j = 1, \dots, n$$

$$z_0 = ?, \quad z_{n+1} = ?$$

Релаксационные осцилляторы

$$\dot{u} = \lambda f(u(t-1))u,$$

$$u(t) > 0,\ \lambda \gg 1,\ h \in (0,1),\ f(u),\ g(u) \in C^1(\mathbb{R}_+),\ \mathbb{R}_+ = \{u \in \mathbb{R} : u \geq 0\},$$

$$f(0) = 1,\ \ f(u) = -a_0 + O(1/u),\ uf'(u) = O(1/u),$$

$$u^2 f''(u) = O(1/u),\ u^2 g''(u) = O(1/u)\ \text{при}\ u \to +\infty, \quad a_0 > 0$$

$$\dot{u}_j = d\,(u_{j+1} - u_j) + \lambda f(u_j(t-1))u_j, \quad j = 1, \dots, m, \quad u_{m+1} = u_1,$$

$d = \mathrm{const} > 0.\ \lambda \gg 1.$

$$u_1 \equiv \ldots \equiv u_m = u_*(t, \lambda),$$

$u_*(t, \lambda)$

$$u_1 = \exp(x/\varepsilon), \quad u_j = \exp\left(x/\varepsilon + \sum_{k=1}^{j-1} y_k\right), \quad j = 2, \dots, m, \quad \varepsilon = 1/\lambda.$$

$$\dot{x} = \varepsilon d\,(\exp y_1 - 1) + F\big(x(t-1),\, \varepsilon\big),$$

$$\dot{y}_j = d\left[\,\exp y_{j+1} - \exp y_j\right] + G_j\big(x(t-1),\, y_1(t-1),\, \dots,\, y_j(t-1),\, \varepsilon\big),$$

$$j = 1, \dots, m-1,$$

$$y_m = -y_1 - y_2 - \ldots - y_{m-1},\; F(x, \varepsilon) = f\big(\exp(x/\varepsilon)\big),$$

$$G_j(x, y_1, \dots, y_j, \varepsilon) = \frac{1}{\varepsilon}\left\{f\left(\exp\left(x/\varepsilon + \sum_{k=1}^{j} y_k\right)\right) - f\left(\exp\left(x/\varepsilon + \sum_{k=1}^{j-1} y_k\right)\right)\right\},$$

$$j = 1, \dots, m-1.$$

$$x_0(t) = \begin{cases} t & \text{при } 0 \le t \le 1, \\ 1 - a(t-1) & \text{при } 1 \le t \le t_0 + 1, \\ -a + t - t_0 - 1 & \text{при } t_0 + 1 \le t \le T_0, \end{cases} \qquad x_0(t + T_0) \equiv x_0(t).$$

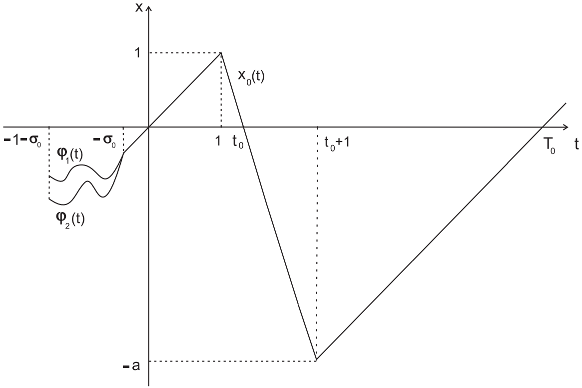

Рис.: 1

$$\dot{y}_j = d\left[\exp y_{j+1} - \exp y_j\right]$$

$$y_j(1+0) = y_j(1-0) - (1+a)\,y_j(0),$$

$$y_j(t_0+1+0) = y_j(t_0+1-0) - (1+1/a)\,y_j(t_0), \quad j = 1,\dots,m-1,$$

$$y_m = -y_1 - y_2 - \ldots - y_{m-1},$$

$$(y_1,\dots,y_{m-1})\Big|_{t=-\sigma_0} = (z_1,\dots,z_{m-1}),$$

$t_0 = 1 + 1/a.$

$$z \to \Phi(z) \stackrel{\text{def}}{=} \left(y_1^0(t,z), y_2^0(t,z), \dots, y_{m-1}^0(t,z)\right)\Big|_{t=T_0-\sigma_0},$$

$z = (\varphi_2(-\sigma_0), \dots, \varphi_m(-\sigma_0))$.
$z = z_*$

$$\varphi_*(t) = \big(\varphi_1^*(t), \dots, \varphi_m^*(t)\big):\ \varphi_1^*(t) = x_0(t),\ \varphi_j^*(t) = y_{j-1}^0(t+T_0, z_*),$$
$$j = 2, \dots, m, \quad -1-\sigma_0 \le t \le -\sigma_0$$

Моделирования химических синапсов

$$\dot u_j = \lambda f(u_j(t-1))u_j + b\, s_{j-1}(u_{j-1})(u_* - u_j), \quad j = 1, \ldots, m,$$

где $u_0 = u_m$, $s_0 = s_m$.

$$\dot u_j = [\lambda f(u_j(t-1)) + b\, g(u_{j-1}) \ln(u_*/u_j)]u_j, \quad j = 1, \ldots, m, \quad u_0 = u_m,$$

$b = \text{const} > 0$, $u_* = \exp(c\,\lambda)$, $c = \text{const} \in \mathbb{R}$, $g(u) \in C^2(\mathbb{R}_+)$

$$g(u) > 0 \quad \forall\, u > 0, \quad g(0) = 0;$$

$$g(u) - 1,\ ug'(u),\ u^2 g''(u) = O(1/u) \ \text{при}\ \ u \to +\infty.$$

$x_j = (1/\lambda)\ln u_j,\ j = 1,\dots,m$

$$\dot x_j = F(x_j(t-1),\varepsilon) + b\,(c - x_j)G(x_{j-1},\varepsilon), \quad j = 1,\dots,m,$$

где $\varepsilon = 1/\lambda \ll 1,\ x_0 = x_m,\ F(x,\varepsilon) = f(\exp(x/\varepsilon)),\ G(x,\varepsilon) = g(\exp(x/\varepsilon))$.

$$\lim_{\varepsilon\to 0} F(x,\varepsilon) = R(x), \quad \lim_{\varepsilon\to 0} G(x,\varepsilon) = H(x),$$

где

$$R(x) = \begin{cases} \ \ 1 & \text{при } x < 0, \\ -a & \text{при } x > 0. \end{cases}, \quad H(x) = \begin{cases} \ \ 0 & \text{при } x < 0, \\ 1 & \text{при } x > 0. \end{cases}$$

$$\dot x_j = R(x_j(t-1)) + b\,(c - x_j)H(x_{j-1}), \quad j = 1,\dots,m, \quad x_0 = x_m.$$

$$x_j = x(t + (j-1)\Delta, \varepsilon), \quad j = 1, \ldots, m,$$

где $\Delta > 0$.
$x(t, \varepsilon)$ – периодическое решение вспомогательного уравнения

$$\dot{x} = F(x(t-1), \varepsilon) + b\,(c - x)G(x(t-\Delta), \varepsilon)$$

периода $T = m\Delta/k,\ k \in \mathbb{N}$.

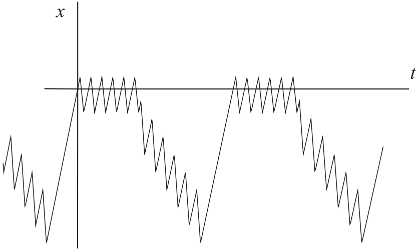

Рис.: 2

Спасибо за внимание!